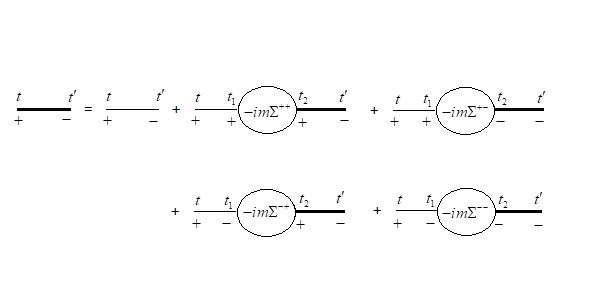
**Self-Energy**

**Self-Energy for GR (possibly time-dependent interactions)**

It seems plausible that the recursive equation for GR, and its concomitant self-energy, can be formulated as is done in the identical multi-particle case. And that everything would carry over straightforwardly. Presuming so, we can examine the self energy concept in the context of the general contour ordered GF, which may include time-dependent interactions. The self energy contains the energy corrections, lifetime corrections, and spectral weight of the excitations in question. It is the same as the information obtained from doing the TDPT in QM1. So consider the self-energy expansion for G--, for instance,



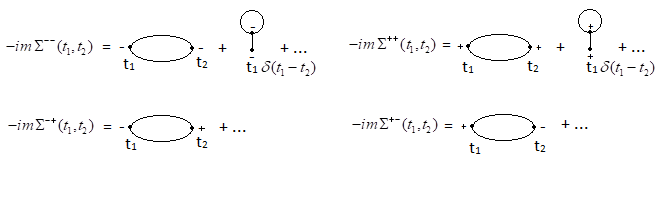
Note I’m defining the bubble as -imΣ to make the final form of the GF come out prettier, although, that means my self energy doesn’t really have units of energy, and it makes other things look uglier. Normal notation is rather to call the bubble -iΣ, not -imΣ, although Σ still wouldn’t have units of energy. This equation would read



which simplifies a little to:



Some of the self-energy terms, for a cubic interaction, would be:



The general requirement is simply that Σab starts with an ‘a’ vertex and ends with a ‘b’ vertex, where ‘a’, and ‘b’ are ±. We can have any kind of vertex in between the end ones though. Note I don’t think external fields are typically compatible with this expansion (don’t see how you could have a recursive equation because an external potential line cannot connect to two GF’s at once). Observe they’d only show up in Σ—and Σ++. And similarly for the other 3 GF. Clearly, we cannot so far write any *1* of the GF in terms of itself and a self-energy. This is the advantage of working with causal GF when we can. And a disadvantage of working with the NEGF. But if we were to write out all 4 of these equations, we would find that we could combine them in matrix form,



with



Just to check, leaving the integrals implicit, we have:



So that works out. Additionally, we know that the free GF’s satisfy the differential equation,



and if we apply this fact to the equation above, then we’d have,



Despite the symmetry of the formulation, it doesn’t take into account the fact that the GF’s are linearly related. To reduce the equations to the only degrees of freedom, we can make the following matrix transformation on the equations. Define:



Then the transformed matrix, G´, is:



Now from the relations in the Formal properties file:



this simplifies to:



where we define F = G-- + G++. The transformed self energies are, Σ´:



To make progress on this, we can use the following relationship:



which will eliminate the lower right element. Let’s prove this before proceeding. So we’ll start with the relationship GC + GAC = G> + G<. And if we insert the self energy equations for GC,AC,>,< into this relationship (leaving integrals implicit) we have:



and then apply the differential operator –(m∂t2 + k) to each side of the equation. And we’ll get, using some notational license:



But now recall from formal properties that:



which implies,



and so the last line simplifies to:



and so we have,



whicch is what we wanted to prove. Okay now let’s go back to the self-energy equation and use this relationship. We’ll have:



where we define Ω = Σ-- + Σ++, ΣR = Σ-- + Σ-+, and ΣA = Σ-- + Σ+-. The usefulness of this formulation lies in the fact that some of the equation decouple. Consider the recursive matrix GF equation again, leaving the integrals implicit.



Inserting our R’s, we have:



And inserting our results for these guys in,



The nice thing about this is that the equations for GA and GR are entirely self-consistent, excepting the fact that their respective self-energies involve other GF’s anyway. So explicitly we have:



Therefore, we *can* write out a self-consistent equation for GA, or GR in terms of just itself. If we operate on both sides of the retarded one, say, with the free particle Schrodinger equation we get,



which gives us an integro-differential equation from which one might attempt a non-perturbative calculation of G. The nice thing about this equation is that it clearly decouples GR from the rest of it, something the exact differential equation in the previous file did not do. Of course, this equation would only be as accurate as the exact coupled one in the limit that we include all terms in Σ. You might think of the integral term as a sort of self consistent single particle potential approximation to the interaction. One could attempt to solve this equation iteratively. For time-independent systems, we can go to Fourier space and obtain,



Inversion of this equation yields,



Recalling,



this comes to:



Given this form, we can obtain the spectral function:



The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole.



Suppose we find the root of this equation ω2 = [we’ll impicitly note that Σ is a function of ω2 because ω only appears in the G’s as squared]. Then we’ll expand the denominator of the Green’s function,



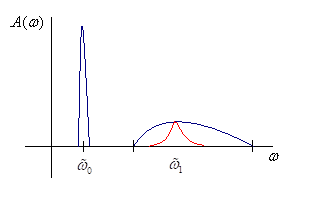
which we can write as:



Z is called the renormalization factor. Z < 1 of course. The closer it is to 1, the closer the approximation is to a real eigenstate. Would the root 0 just be the frequency of oscillation about the new minimum? I guess not entirely because there could be an imaginary part as well. The spectral function will come to:



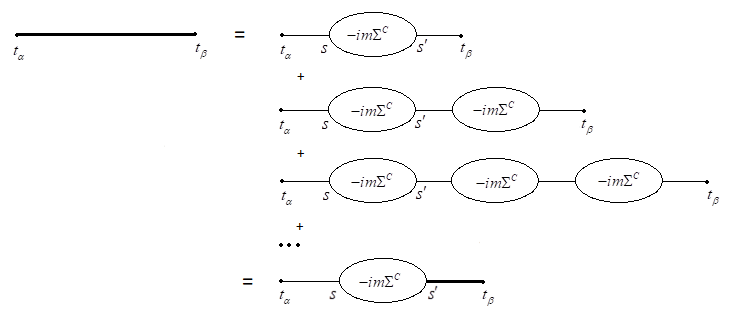
and again, the identifications make sense. Note how ImΣR must be less than 0 for A to be positive. A typical spectral function, A(ω), will look like below (this would be before we made the approximation about a single pole that we just did), in blue.



The presence of two peaks indicates that there are two excitations of the system – could be that our potential is of such a form that it has two minima, instead of the single one our unperturbed system has. The width of the peak is given by the ImΣ. So the delta function thing on the left would have that ImΣ = 0, while not so for the right hand curve. If we tried to approximate the latter with a Lorentzian, as we do above, we’d have the red curve, whose location gives the energy best associated with it, and whose width would give the scattering rate basically, and whose height would roughly give the renormalization factor, basically the amount that this would resemble a quasi-particle.

**Self-Energy for GC (time-independent interaction)**

The former analysis applies for time-independent interactions too of course. But I’ll do it differently here. So for time-independent interactions we *can* do everything in terms of GC alone. Defining the self energy bubble as -imΣC (basically -imΣ—but with only – internal vertices), we can write:



which is equivalent to the equation:



Since the interaction is time-independent, ΣC should depend only on the difference s – s´. This would allow a temporal Fourier transform on our recursive equation. And we’d have:



And this gives us:



This allows us to identify the self-energy term as a correction to the non-interacting energy, taking account of the perturbation. To the best degree possible it provides a description of the energy of GS within the ‘independent harmonic oscillations’ model, giving its new energy/oscillation frequency, and lifetime. And of course we would add all of the self energy terms, not just the one bubble we have here. I would tend to think that it’s equivalent to BW perturbation theory? Not sure. Now let’s talk about the analytic properties of the self energy. The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole.



Suppose we find the root of this equation ω2 = [we’ll impicitly note that Σ is a function of ω2 because ω only appears in the G’s as squared]. Then we’ll expand the denominator of the Green’s function,



which we can write as:



Z is called the renormalization factor. Z < 1 of course. The closer it is to 1, the closer the approximation is to a real eigenstate. Would the root 0 just be the frequency of oscillation about the new minimum? I guess not entirely because there could be an imaginary part as well.